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Some remarks on oscillating inflation

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In a recent paper Damour and Mukhanov described a scenario where inflation may continue during the oscillatory phase. This effect is possible because the scalar field spends a significant fraction of each period of oscillation on the upper part of the potential. Such an additional period of inflation could push perturbations after the slow roll regime to observable scales. Although in this work we show that the small region of the Damour-Mukhanov parameter q gives the main contribution to oscillating inflation, it was not satisfactorily understood until now. Furthermore, it gives an expression for the energy density spectrum of perturbations, which is well behaved in the whole physical range of q.

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I. INTRODUCTION

Nowadays inflation is a widely accepted element of early cosmology [1]. It gives the possibility of solving many of the shortcomings of the standard hot big bang model and provides the source for the early energy density fluctuations responsible of the large scale structure of the universe observed today. Although there are many models of inflation, the underlying physical ideas are well established. These are characterized by a period of ''slow roll'' evolution of a scalar field (called inflaton) toward the vacuum potential. During this period the field changes very slowly, so that the kinetic energy $\dot{\varphi}^2/2$ remains smaller than its potential energy $V(\varphi)$. The energy density associated with the scalar field acts as a ''cosmological constant'' term, allowing a period of quasiexponential expansion of the scale factor. When the period of inflation ends, the scalar field φ start a phase of rapid coherent oscillations around the vacuum.

Very recently [2,3] it has been pointed out that inflation can persist during the coherent oscillations of the inflation field phase. This exciting result is possible when the inflaton potential verifies a simple constrain of curvature far from the core convex part, where the inflaton field can roll slowly. The efficiency of this phenomena could have important implications for grand unified theory (GUT) scale baryogenesis [4]. In fact, as suggested by Damour and Mukhanov [2], it can be expected that due to the increase of the oscillation frequency, there is the possibility to generate massive particles heavier than $\sim 10^{16}$ GeV.

In Ref. [2] Damour and Mukhanov estimated the amount of inflation to be ~ 10 e-fold (powers of the scale factor). They argue that this effect can be more efficient than the parametric resonance effect [5] for the amplification of cosmological perturbations [6]. In Ref. [3] Liddle and Mazumdar showed that Mukhanov *et al.* overestimated the number of e-fold because they have used a slow-roll definition of this object. In their paper, Liddle and Mazumdar found an analytical expression for the number of e-fold of inflation using the appropriate definition finding a number of ~ 3 e-fold concluding that this effect is not very efficient. The study of adiabatic perturbations in this phase has been made by Taruya [7]. He found a poor amplification in the case of a single scalar field model but anticipated an enormous amplification for multifield systems.

In this paper we review the problem. In particular we find that the analytical expressions used to compare with the numerical estimation are not well defined in the $q \sim 0$ region and propose a way to correct these analytical estimations. Furthermore, with this result we study the evolution of the scalar field finding total agreement with the conclusions of Ref. [3] for $q > 0.2$, but a remarkable different result for small q. For this region, the initial conditions are very important. We find that $q \sim 0$ gives the leading contribution for oscillating inflation and the dominant part in the amplification of the fluctuations.

The paper is organized as follows. First we describe briefly the Damour-Mukhanov model. Then, we make some comments about the initial conditions for this phenomenon and later we propose an improved expression, valid for the leading region of q, which is our main contribution.

II. BASIC EQUATIONS

Now we shall restrict ourselves to models of inflation driven by a single scalar field. The equations are

$$\ddot{\varphi} + 3H\dot{\varphi} + V,_{\varphi} = 0, \tag{1}$$

$$H^2 = \kappa^2 \left(\frac{1}{2}\dot{\varphi}^2 + V \right). \tag{2}$$

Here $H = \dot{a}/a$ is the Hubble parameter, a is the scale factor of the universe and $\kappa^2 = 8\pi/3M_p^2$ with $M_p = 1.2\times 10^{19}$ GeV the Planck mass. During the oscillatory phase of φ we have two time scales; the inverse of the frequency ω^{-1} of oscillations of φ and the inverse of the rate of expansion H^{-1}. If the limit $\omega \gg H$ is taken we can neglect terms proportional to H in the equations. So from Eq. (1) we can integrate to obtain

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$$\rho=\frac{1}{2}\dot{\varphi}^2+V=cte=V_m, \qquad (3)$$

where $V_m=V(\varphi_m)$ is the maximum value of $V(\varphi)$ in each oscillation when the field reaches the maximum value φ_m. From this relation we obtain the period of a single oscillation, $T=4\int_0^{\varphi_m}d\varphi[2(V_m-V(\varphi))]^{1/2}$. When $\omega\ll H$ we can define an adiabatic average index γ by $\gamma=\langle(\rho+p)/\rho\rangle$, where the bracket means $\langle a\ \dots\rangle=T^{-1}\int_0^T\dots dt$. Equations (1),(2) can be rewritten in the fluid form

$$\dot{\rho}=-3H(p+\rho), \qquad (4)$$

$$\frac{\ddot{a}}{a}=-\frac{1}{3}(\rho+3p), \qquad (5)$$

then from the definition of γ and Eqs. (4),(5) we have several ways to compute the adiabatic index

$$\gamma=\frac{\langle\dot{\varphi}^2\rangle}{V_m}=\frac{\langle\varphi V_{,\varphi}\rangle}{V_m}=2\left(1-\frac{\langle V\rangle}{V_m}\right). \qquad (6)$$

Because $p=(\gamma-1)\rho$ and Eq. (5) we have a superluminal expansion $\ddot{a}>0$ when $\gamma<2/3$. From the last two relations in Eq. (6) the inequality $\gamma<2/3$ leads to

$$\langle V-\varphi V_{,\varphi}\rangle>0. \qquad (7)$$

III. THE DAMOUR-MUKHANOV MODEL

Until now everything has been done for an arbitrary potential, but from now on we shall consider the potential

$$V(\varphi)=\frac{A}{q}\left[\left(\frac{\varphi^2}{\varphi_c^2}+1\right)^{q/2}-1\right], \qquad (8)$$

where q is a dimensionless parameter, $A=[\text{mass}]^4$ is a constant and $\varphi_c=[\text{mass}]$ determines the size of the convex core of $V(\varphi)$. We assume for a while that φ_c marks the end of *oscillating inflation*. The analysis made in Ref. [2] works well far from the core of the potential. Further, the limit $\varphi\gg\varphi_c$ of Eq. (8) was written as

$$V(\varphi)\simeq\frac{A}{q}\left(\frac{\varphi}{\varphi_c}\right)^q. \qquad (9)$$

In this case, the adiabatic index can be computed exactly given [8] by

$$\gamma=\frac{2q}{q+2}, \qquad (10)$$

so, from the inequality $\gamma<2/3$ we note that to hold inflation during the oscillatory phase we must have $q<1$. By using Eq. (10) in Eq. (4) we obtain $\dot{\rho}=-3H\gamma\rho$ and together with Eq. (2) we have

$$a\propto t^{2/3\gamma}=t^{(q+2)/3q}, \qquad (11)$$

$$\varphi_m\propto t^{-2/q}\propto a^{-6/(q+2)}, \qquad (12)$$

$$\rho=V(\varphi_m)\propto t^{-2}\propto a^{-6q/(q+2)}, \qquad (13)$$

where φ_m is the amplitude of the oscillations, $\varphi_c<\varphi_m<\varphi_s$ and φ_s is a typical value of φ at the end of slow-roll inflation and the beginning of oscillating inflation. To compute the number of e-fold of inflation during oscillating inflation we cannot use the standard expression $N=\ln(a_f/a_i)$, appropriate for the slow-roll stage, but the improved expression proposed in Ref. [3]

$$\tilde{N}=\ln\frac{a_fH_f}{a_iH_i}, \qquad (14)$$

because in each oscillation, while the field spends time in the core region, the universe continue their expansion so H can vary. Then from Eqs. (2) and (13) $H\propto a^{-3q/(q+2)}$ the product $aH\propto\varphi_m^{(1-q)/3}$ and from Eq. (14) we obtain

$$\tilde{N}\simeq\frac{1-q}{3}\left[\ln\frac{qM_p}{\varphi_c}-2\right], \qquad (15)$$

where we have used $\varphi_s\sim qM_p/\sqrt{16}\pi$. In [3] the numerical curves for $\varphi_c=10^{-6}M_p$ show that $\tilde{N}\lesssim3$. Using the analytical expression (15) we do not find agreement for small values of q. However there is not a compelling reason to believe in Eq. (15) for small q.

IV. THE SMALL q REGION

To study the small q region, we must use the correct limit $q\to0$ of Eq. (8) which leads to

$$V(\varphi)\simeq\frac{A}{2}\ln\left[\left(\frac{\varphi}{\varphi_c}\right)^2+1\right], \qquad (16)$$

so, if now we take the limit $\varphi\gg\varphi_c$ we obtain the logarithmic potential $V(\varphi)\simeq A\ln(\varphi/\varphi_c)$. A very important fact to note from Eq. (9) is that the limit $q\to0$ does not exist. Of course, the expression (9) is wrong around the $q\sim0$ region and the expressions derived from this are ill-defined. But, some work has been done in this regard [2]. For the logarithmic potential the adiabatic index is $\gamma=1/\ln(\varphi_m/\varphi_c)$, so from Eqs. (4) and (2) we obtain

$$a(t)\propto\exp\left[-\frac{A}{2}(t_{end}-t)^2\right], \qquad (17)$$

but this form does not permit us to write an explicit expression for $\tilde{N}$ (see Ref. [3]). Let us make some comments about this result. Because $\gamma=1/\ln(\varphi_m/\varphi_c)$ from Eq. (4) we obtain $\dot{\rho}=-3HA$, then $a\sim(\varphi_m/\varphi_c)^{-1/3}$. Moreover, from Eq. (2) we have $H\propto\rho^{1/2}\propto(\ln(\varphi_m/\varphi_c))^{1/2}$; then to compute $\tilde{N}$ we should evaluate the factor $(\ln(\varphi_m/\varphi_c))^{1/2}(\varphi_m/\varphi_c)^{-1/3}$ at the extremes φ_s and φ_c, but this is not possible. The problem arises when φ_m is chosen close to φ_c in a expression valid for $\varphi\gg\varphi_c$. Because Eq. (9) is valid for $\varphi\gg\varphi_c$ too, the same problem should appear in the calculation of $\tilde{N}$. In fact this is

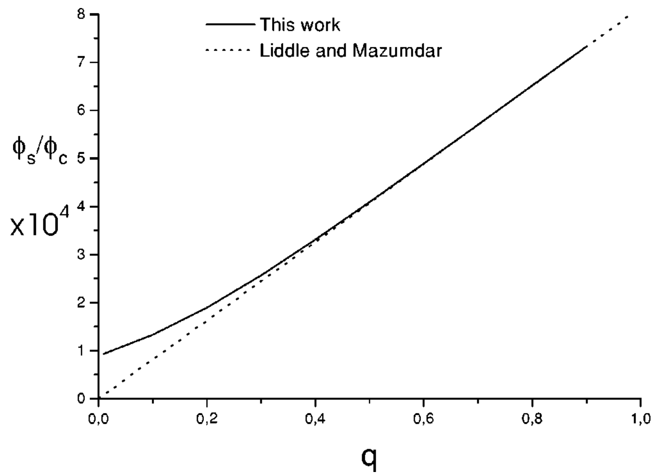

FIG. 1. We plot the q dependence of φ_s. We see that for $q > 0.2$ both curves agree but for smaller values of q the field φ_s grows, preventing the fall of $\tilde{N}$.

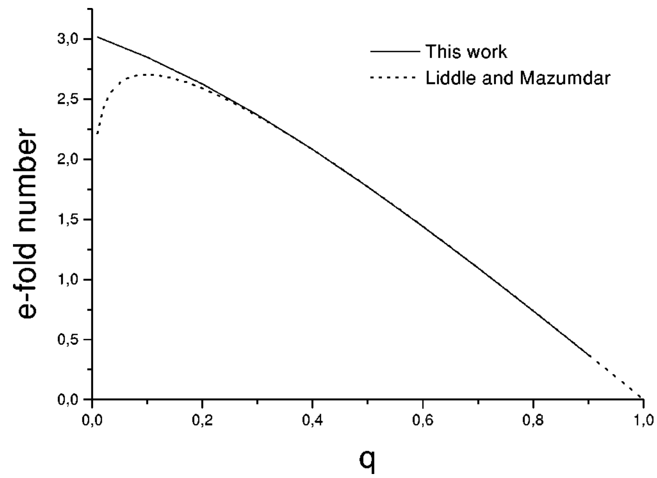

FIG. 2. The e-fold number $\tilde{N}$ is shown as a function of q, taking into account the behavior of φ_s described for Eq. (20).

the case but, to see that, we must include the constant term A/q in Eq. (9) or equivalently, write the limit correctly.

When we use the potential

$$V(\varphi) \simeq \frac{A}{q}\left[\left(\frac{\varphi}{\varphi_c}\right)^q - 1\right], \tag{18}$$

the adiabatic index become time-dependent, satisfying the equation

$$\gamma\rho = \frac{2q}{q+2}\left(\rho + \frac{A}{q}\right), \tag{19}$$

where $\rho(t) = V(\varphi_m(t)) = A[(\varphi_m(t)/\varphi_c)^q - 1]/q$. Replacing this in Eq. (4) we obtain the same behavior as in Eq. (12). But when we calculate H we obtain $H \propto \rho^{1/2} \propto q^{-1/2}[(\varphi_m/\varphi_c)^q - 1]^{1/2}$, which is not well defined at the point $\varphi_m = \varphi_c$. The same happens to the e-fold number, as mentioned before.

The authors of [3] found $\varphi_s \sim q M_p/\sqrt{16\pi}$ using the potential (9). From this result they found a decrease of $\tilde{N}$ at decreasing values of q. This seems very strange because, for smaller values of q we obtain flatter potentials at $\varphi \gg \varphi_c$, then φ rolls slowly most of the time increasing the amount of inflation. Furthermore, we are not safe of how they set the initial conditions for φ in their numerical analysis. In general, the initial and final field configuration depends on the potential.

For q close to cero, φ_s is not proportional to q. In fact, we know that φ_s came from the saturation of the slow-roll inequality $|V'/V| < \sqrt{48\pi}/M_p$ using the expression (18) for the potential

$$\left(\frac{\varphi_s}{\varphi_c}\right)^{q-1} = \frac{\sqrt{48\pi}\varphi_c}{q M_p}\left[\left(\frac{\varphi_s}{\varphi_c}\right)^q - 1\right] \tag{20}$$

for the $\varphi \gg \varphi_c$ region. In Fig. 1 we see the behavior of φ_s in terms of q given by Eq. (20). Of course, in the large q-region both curves agree. In order to illustrate this point and compare with Ref. [3] we use its expression given by Eq. (14) but instead of using $\varphi_s \sim q M_p/\sqrt{16\pi}$ we use the numerical values of φ_s obtained from Eq. (20). The results are plotted in Fig. 2. In the small q-region the field φ_s grows preventing the fall of $\tilde{N}$ predicted in Ref. [3].

Moreover, as we have anticipated before, the value of the field at the end of oscillating inflation will have a q-dependence too. We know from Ref. [2] that the intercept $U(\varphi) = V(\varphi) - \varphi V_{,\varphi}$, must be positive to hold oscillating inflation. Let us define φ_f to be the value of the inflaton field φ at which $U(\varphi_f) = 0$. This condition represents the end of inflation due to oscillation. We need thus to compare φ_f for different values of q.

If we take the potential (18) and define $x = \varphi/\varphi_c$, we obtain

$$U(x) = \frac{A}{q}[x^q(1-q) - 1]. \tag{21}$$

From this equation we can extract a explicit expression for φ_f. If we impose $U(x_f) = 0$ we obtain the value of the scalar inflaton field at the end of this phase

$$\varphi_f = \varphi_c(1-q)^{-1/q}. \tag{22}$$

Using the improved expression Eq. (14) for the e-fold number (Ref. [3]), but inserting the corrected values for φ_s and φ_f given by Eqs. (20) and (22) we obtain

$$\tilde{N} \simeq \ln\left\{\left(\frac{\varphi_s}{\varphi_f}\right)^{(2+q)/6}\left[\frac{(\varphi_f/\varphi_c)^q - 1}{(\varphi_s/\varphi_c)^q - 1}\right]^{1/2}\right\}. \tag{23}$$

The corrected value for φ_f leads to an even smaller amount of inflation when comparing with the value obtained in Ref. [3]. In Fig. 3, we plot Eq. (23) and show the behavior of both effects combined. Because φ_f is greater than φ_c, the amount of inflation is smaller than one obtained by Liddle *et al.* [3] in the whole range of $q \in (0,1)$. Moreover, the correct values of φ_s produce a positive contribution to $\tilde{N}$ in the small

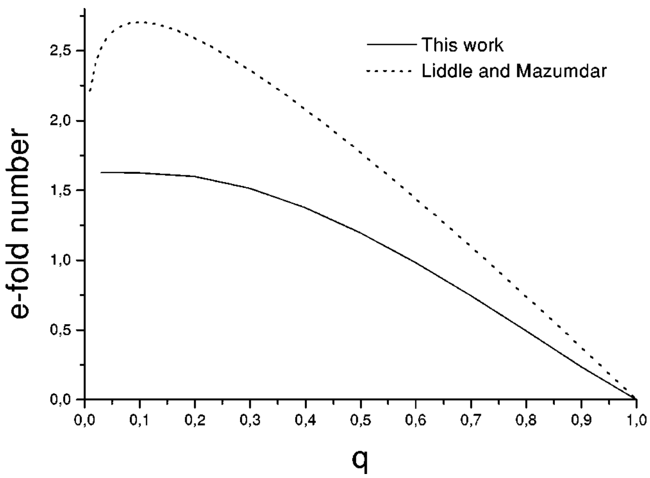

FIG. 3. The e-fold number $\tilde{N}$ is plotted vs q, taking into account the combined effects: the behavior of φ_s described by Eq. (20) and the definition of φ_f discussed in the text.

q-range, which avoids the fall of $\tilde{N}$, as q goes to smaller values, predicted by Liddle *et al.* [3]. Again, it is not possible to show the whole range of q because $q > \varphi_c/\varphi$ [see comments below Eq. (20)].

Because oscillating inflation adds e-folds of inflation after the slow-roll regime, where the observed perturbations are generated, it is possible that this additional period of inflation could push perturbations to observable scales. In order to obtain the required amplitude of density perturbations and without imposing unphysical constraint on the potential, we should compute the density perturbation spectrum for the model being studied.

In Ref. [3] an expression for this object was derived:

$$\delta_H^2 = \frac{512\pi}{75}\frac{A}{q^3 M_p^6}\frac{\varphi^{q+2}}{\varphi_c^q}.$$

However, this expression is not well defined close enough to zero. Using the primordial density perturbation spectrum δ_H as was define in [3] we obtain, for Eq. (18),

$$\delta_H^2 = \frac{512\pi}{75}\frac{A}{q^3 M_p^6}\left\{\frac{[(\varphi/\varphi_c)^q - 1]^3}{(\varphi/\varphi_c)^{2q}}\right\}, \tag{24}$$

which is well defined even for the small values of q:

$$\delta_H^2 \approx \frac{512\pi}{75}\frac{A}{M_p^6}\ln^3\left(\frac{\varphi}{\varphi_c}\right). \tag{25}$$

Because the Cosmic Background Explorer (COBE) satellite requires $\delta_H \approx 2\times 10^{-5}$, in the $q=0$ case the amplitude of the potential for $\varphi_c = 10^{-6}M_p$ gives $A^{1/4} \sim 2\times 10^{-3}M_p$, which is a typical number for inflationary models.

V. SUMMARY

As a summary, we have made some corrections about how to compute the e-fold number, which accounts for the amount of inflation during the oscillatory phase. In particular we note that previous studies are not accurate because they are not valid close to the core of the potential $\varphi \sim \varphi_c$. Thus, we make the analysis for the small q region of the potential, which has not been considered until now. Finally we find that, in order to extract the correct amount of inflation during this phase, a very careful definition of initial and final field configuration is needed. Our results show that near $q \sim 0$ the e-fold number is maximal but it is still not enough to be more efficient than the parametric resonant effect discussed in [5].

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[1] A. H. Guth, Phys. Rev. D **23**, 347 (1981); A. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, New York, 1990); E. Kolb and M. Turner, *The Early Universe* (Addison-Wesley, Reading, 1990).

[2] T. Damour and V. Mukhanov, Phys. Rev. Lett. **80**, 3440 (1998).

[3] A. Liddle and A. Mazumdar, Phys. Rev. D **58**, 083508 (1998).

[4] E. Kolb, A. Linde, and A. Riotto, Phys. Rev. Lett. **77**, 4290 (1996).

[5] J. Traschen and R. Brandenberger, Phys. Rev. D **42**, 2491 (1990); Yu. Shtanov, J. Traschen, and R. Brandenberger, *ibid.* **51**, 5438 (1995); L. Kofman, A. Linde, and A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994); **76**, 1011 (1996); Phys. Rev. D **56**, 3258 (1997).

[6] F. Finelli and R. Brandenberger, Phys. Rev. Lett. **82**, 1362 (1999).

[7] A. Taruya, Phys. Rev. D **59**, 103505 (1999).

[8] M. Turner, Phys. Rev. D **28**, 1243 (1983).